Computer Vision & Digital Image Processing

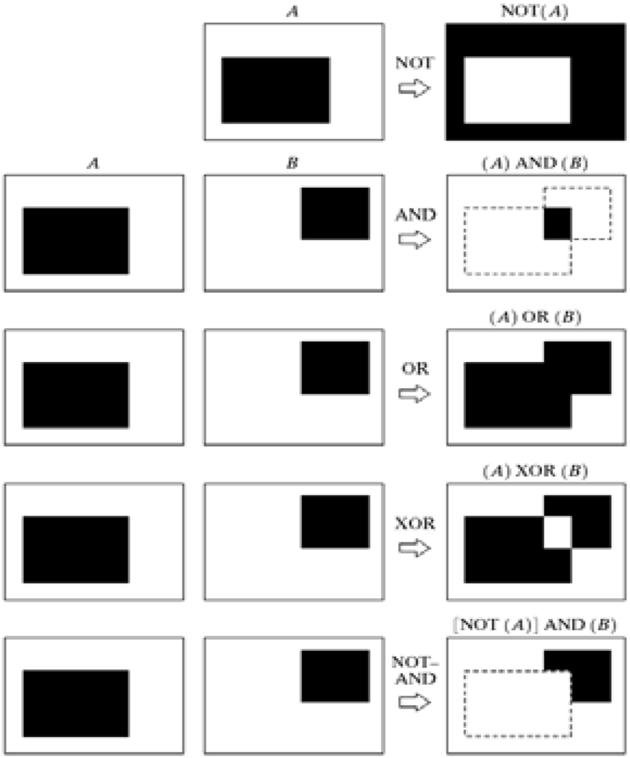
Morphological Image Processing

Introduction

- *Morphology* – a branch of biology concerned with the form and structure of plants and animals
- *Mathematical morphology* – a tool for extracting image components useful in the representation and description of image shape including:
  - Boundaries
  - Skeletons
  - Convex hull
- We will also look at morphological techniques for
  - Filtering
  - Thinning
  - Pruning

Logical Operations Involving Binary Images

- Principal logic operations
  - AND
  - OR
  - NOT (COMPLEMENT)
- Functionally complete
  - Combined to form any other logic operation
- Logic operations described have a one-to-one correspondence with the set operations intersection, union and complement
  - Logic operations are restricted to binary images (not the case for general set operations)



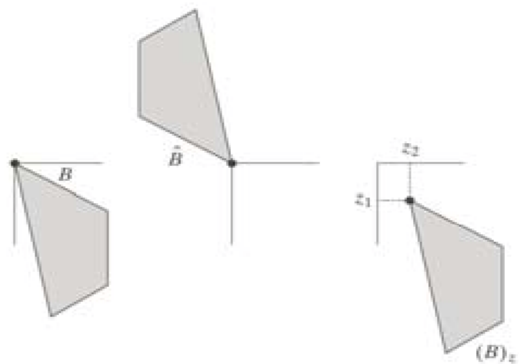
Basic Concepts from Set Theory

- The *reflection* of set B is defined as

$$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$$

- The *translation* of set B by point $z = (z_1, z_2)$ is defined as

$$(B)_z = \{c \mid c = b + z, \text{ for } b \in B\}$$



a b c

FIGURE 9.1
(a) A set, (b) its reflection, and (c) its translation by z.

Structuring Elements

- Set reflection and translation are used extensively in morphological operations based on *structuring elements* (SE)
- An SE is a small set (or subimage) used to “probe” an area of interest for certain properties
  - May be of arbitrary shape and size
  - In practice an SE is generally a regular geometric shape (square, rectangle, diamond, etc.)
  - Generally padded to a rectangular array for image processing

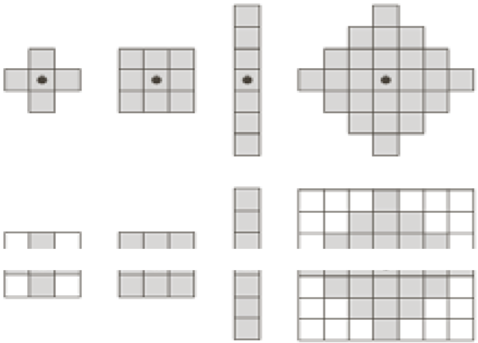
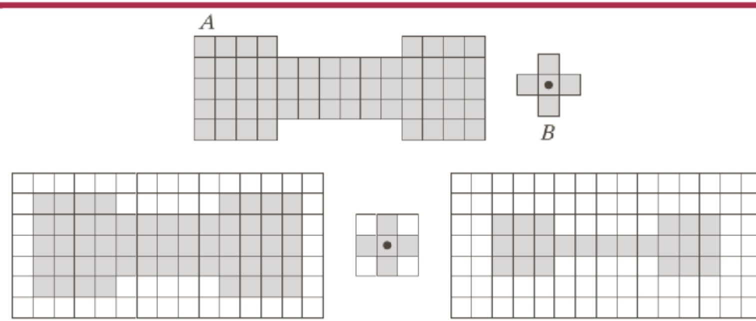


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

Structuring Elements (continued)

- Two-dimensional, or flat, structuring elements consist of a matrix of 0's and 1's, typically much smaller than the image being processed.
- The center pixel of the structuring element, called the origin, identifies the pixel of interest--the pixel being processed.
- The pixels in the structuring element containing 1's define the neighborhood of the structuring element.

Operation with a Structuring Element (example)



a b
c d e

FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

Dilation

- With A and B as sets in Z^2, the *dilation* of A by B, denoted $A \oplus B$, is defined as

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$$

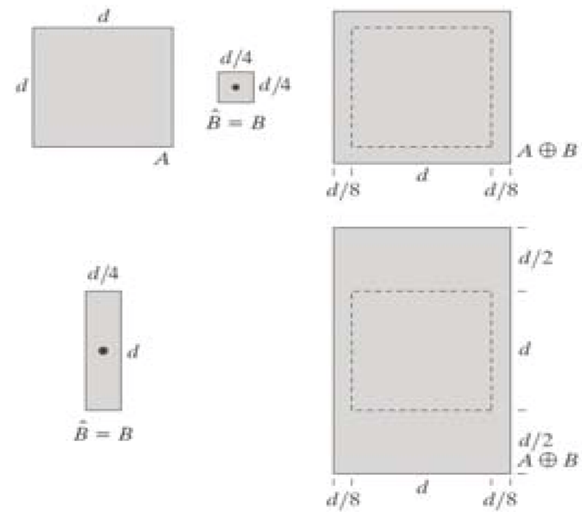
- This formulation is based on the reflection of B about its origin and shifting this reflection by z
- The dilation of A by B is the set of all *displacements*, z, such that $\hat{B}$ and A overlap by at least one element
- Therefore, another expression for the *dilation* of A by B is

$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

- Set B is the *structuring element*

Dilation Example

- $B = \hat{B}$ because B is symmetric with respect to its origin
- The dashed line shows the original set A and the solid boundary shows the limit beyond which any further displacements of the origin of $\hat{B}$ by z would cause the intersection of $\hat{B}$ and A to be empty
- All points inside this boundary constitute the dilation of A by B
- The second case shows more dilation vertically than horizontally

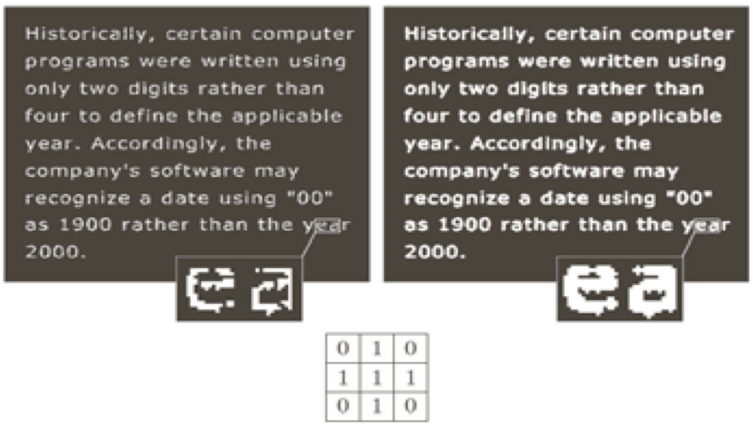


a b c
d e

FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

Dilation Application

- One simple application of dilation is for bridging gaps
- In the image below, the maximum break length is two pixels
- Although low pass filtering can be used to accomplish the same task, this generates a gray-scale image that must then be thresholded to produce a resulting binary image



a b c

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Erosion

- With A and B as sets in Z^2, the *erosion* of A by B, denoted $A \ominus B$, is defined as

$$A \ominus B = \{z \,/\, (B)_z \subseteq A\}$$

- In words, the erosion of A by B is the set of all points z such that B, translated by z, is contained in A
- Dilation and erosion are duals of each other with respect to set complementation and reflection
- Therefore

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

and

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

Erosion Example

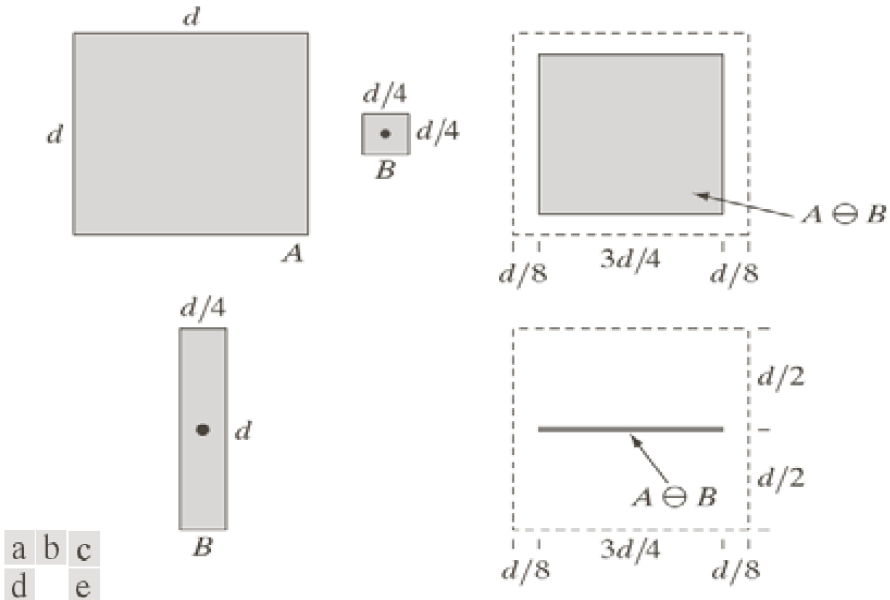


FIGURE 9.4 (a) Set *A*. (b) Square structuring element, *B*. (c) Erosion of *A* by *B*, shown shaded. (d) Elongated structuring element. (e) Erosion of *A* by *B* using this element. The dotted border in (c) and (e) is the boundary of set *A*, shown only for reference.

Erosion and Dilation Application

- One simple application of erosion is for eliminating irrelevant detail (in terms of size) from a binary image
- Note: In general, dilation does not restore fully the eroded objects

Erosion of a binary image with a 13x13 size structuring element and subsequent dilation of the result with the same element.

Opening and Closing

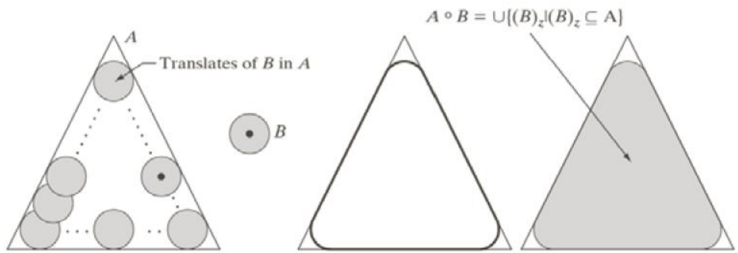
- Morphological *opening* generally
  - Smoothes the contour of an object
  - Breaks narrow isthmuses
  - Eliminates thin protrusions
- Morphological *closing* generally
  - Smoothes contour sections
  - Fuses narrow breaks and long thins gulfs
  - Eliminates small holes
  - Fills gaps in a contour

Opening and Closing

- The opening of set A by structuring element B, denoted $A \circ B$, is defined as
  - $A \circ B = (A \ominus B) \oplus B$
- Thus, opening is defined as the erosion of A by B followed by a dilation of the result by B
- The closing of set A by structuring element B, denoted $A \bullet B$, is defined as
  - $A \bullet B = (A \oplus B) \ominus B$
- Thus, opening is defined as the dilation of A by B followed by a erosion of the result by B

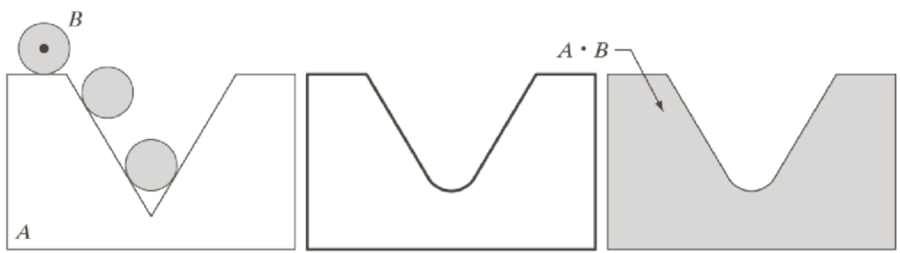
Opening: Geometric Interpretation

- Suppose the structuring element, *B*, is viewed as a rolling ball
- The boundary of $A \circ B$ is established by all points in *B* that reach the *farthest* into the boundary of *A* as *B* is rolled about the *inside* of this boundary
- The opening of *A* by *B* is obtained by taking the union of all translates of *B* that fit into *A*
  - $A \circ B = \cup \{(B)_z \,/\, (B)_z \subseteq A\}$

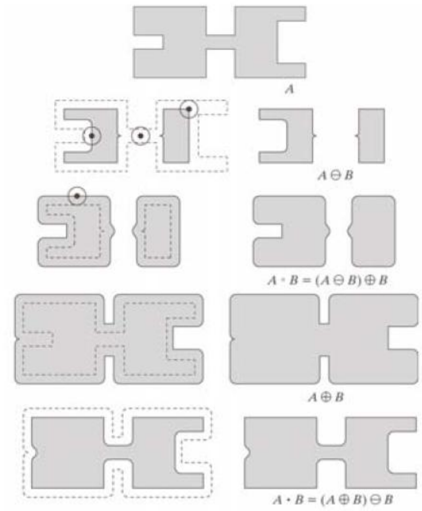


Closing: Geometric Interpretation

- Suppose the structuring element, *B*, is viewed as a rolling ball
- The boundary of $A \bullet B$ is established by all points in *B* that reach the *closest* to the boundary of *A* as *B* is rolled about the *outside* of this boundary
  - $A \bullet B = \{w \mid (B)_z \cap A \neq \varnothing$ for any translate of $(B)_z$ containing $w\}$



Opening and Closing Examples



a	
b	c
d	e
f	g
h	i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Morphological Filtering

- Morphological operators can be used to construct filters similar in concept to spatial filters
- If the filtering objective in question is to eliminate noise and distort data of interest as little as possible, then a morphological filter consisting of an opening followed by a closing can be used
- Recall from the definitions of opening and closing that the more primitive operations of erosion and dilation are used
  - Opening
  - $A \circ B = (A \ominus B) \oplus B$

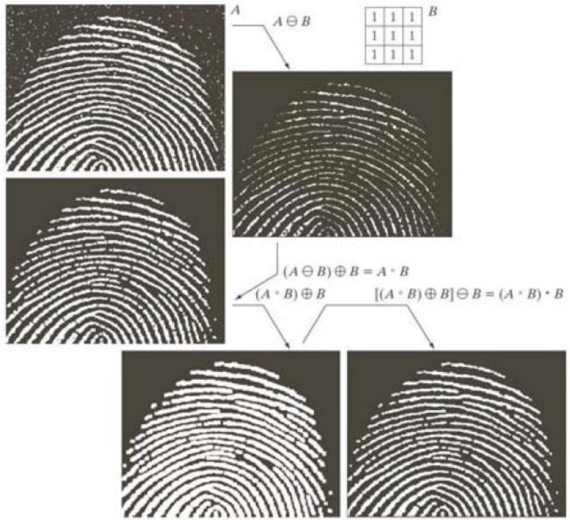
    (erode: $\ominus$; dilate: $\oplus$)

  Closing

  $A \bullet B = (A \oplus B) \ominus B$

  (dilate: $\oplus$; erode: $\ominus$)

Morphological Filtering Example

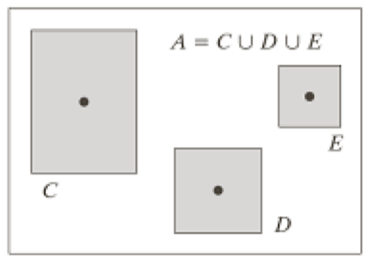


a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

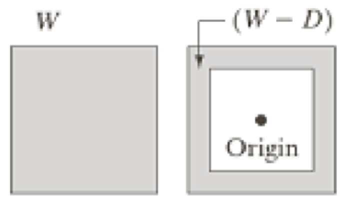
Hit-or-Miss Transform

- The morphological hit-or-miss transform is a basic tool for shape detection
- The basic intent is to find the location of a known shape within a set of shapes
- Assume a set *A* consists of a set of shapes (subsets) *C*, *D*, and *E*
- It is desired to find the location of one of the shapes, *D*
- Let the origin of each shape be its center of gravity



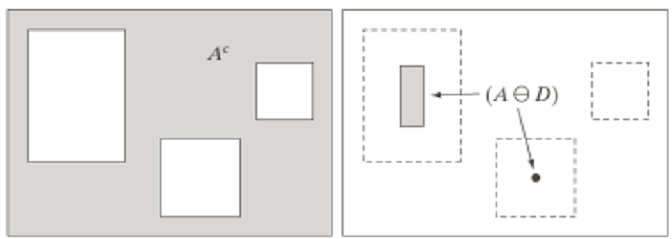
Hit-or-Miss Transform

- Let *D* be enclosed by a small window, *W*
- The *local background* of *D* with respect to *W* is defined as the set difference (*W-D*)



Hit-or-Miss Transform

- The complement of *A*, A^c, is needed in the transform operation
- Let *A* be eroded by *D*
- The erosion of *A* by *D* is the set of locations of the *origin* of *D*, such that *D* is completely contained in *A*
- Viewed geometrically, this is the set of all locations of the origin of *D* at which *D* found a match (hit) in *A*



Hit-or-Miss Transform

- Erode the complement of A, A^c, by the local background set ($W-D$)
- If we now compute the intersection of the two computed values, this give use the location of D
- If B denotes the set composed of D and its background, the match (or set of matches) of B in A, denoted $A \circledast B$, is

- $A \circledast B = (A \ominus D) \cap [A^c \ominus (W-D)]$